

Spin Transfer Without Spin Conservation

Alvaro S. Núñez and A.H. MacDonald

Physics Department, University of Texas at Austin, Austin TX 78712

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We propose a general theory of the spin-transfer effects that occur when current flows through inhomogeneous magnetic systems. Our theory does not rest on an appeal to conservation of total spin, can assess whether or not current-induced magnetization precession and switching in a particular geometry will occur coherently, and can estimate the efficacy of spin-transfer when spin-orbit interactions are present. We illustrate our theory by applying it to a toy-model two-dimensional-electron-gas ferromagnet with Rashba spin-orbit interactions.

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Introduction: The transfer[1, 2] of magnetization from quasiparticles to collective degrees of freedom in transition metal ferromagnets has received attention recently because of experimental[3, 4, 5, 6, 7, 8] and theoretical[9] progress that has motivated basic science interest in this many-electron phenomenon, and because of the possibility that the effect might prove to be a useful way to write magnetic information. The key theoretical ideas that underly this effect were proposed some time ago[1, 2] and rest heavily on bookkeeping which follows the flow of spin-angular momentum through the system. Recent advances in nanomagnetism have made it possible to compare these ideas with experimental observations and explore them more fully. In this Letter, we propose a general theory of spin transfer that does not rest on an appeal to conservation of total spin, focusing instead on the change in the exchange-field experienced by quasiparticles in the presence of non-zero transport currents. Our approach can assess whether or not the current-driven magnetization dynamics in a particular geometry will be coherent, and can predict the efficacy of spin-transfer when spin-orbit interactions are present. It can be formulated within any time-dependent mean field theory of a metallic ferromagnet but is, for transition metals, most appropriately placed in the framework of *ab initio* spin-density-functional theory[10] (SDFT) which is accurate for these systems.

Our picture of spin-transfer is summarized schematically in Fig.[1]. In SDFT, order in a metallic ferromagnet is characterized by excess occupation of *majority*-spin orbitals, at a band energy cost smaller than the exchange-correlation energy gain. (Adopting the common terminology of magnetism, we refer to the scalar and spin exchange-correlation fields of SDFT below as scalar and exchange potentials.) In the ordered state, majority and minority spin quasiparticles are brought into equilibrium by an exchange field that is approximately proportional to the magnetization magnitude and points in the majority-spin direction. The spin-orientation of the singly occupied majority-spin orbitals is the collective-coordinate, the magnetization orientation, that plays the lead role in most magnetic phenom-

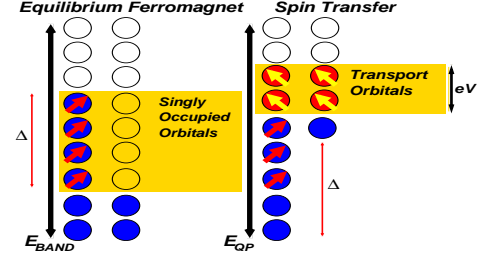


FIG. 1: Left panel: Ground state of a metallic ferromagnet. The low-energy collective degree of freedom is the spin-orientation of singly occupied orbitals. Right panel: Quasiparticles experience a strong exchange field Δ that brings majority and minority spins into equilibrium. Because this field is parallel to the magnetization it does not produce a torque. In an inhomogeneous ferromagnet, the spin orientation of the transport orbitals in a window of width eV at the Fermi energy can differ from the magnetization orientation. The spin-transfer torque is produced by the transport-orbital contribution to the exchange field.

ena. The non-equilibrium current-carrying state of a ferromagnetic metal thin film can then be described using a scattering or non-equilibrium Greens function formulation of transport theory[11]. The current is due to electrons in a narrow *transport window* with width eV centered on the Fermi energy, and can be evaluated by solving the quasiparticle Schroedinger equation for electrons incident from the high-potential-energy side of the film. The spin-transfer effect occurs when the spin-polarization of these transport electrons is not parallel to the magnetization, producing a transport induced exchange field around which the magnetization precesses. We expand on this picture below and illustrate its utility by applying it to a toy-model two-dimensional ferromagnet with Rashba[12] spin-orbit interactions.

Quasiparticle Spin Dynamics: We start by considering single-particle Hamiltonians of the form

$$\mathcal{H} = \frac{p^2}{2m} + V(\vec{r}) - \frac{1}{2}\vec{\Delta}(\vec{r}) \cdot \vec{\tau}, \quad (1)$$

where $V(\vec{r})$ and $\vec{\Delta}(\vec{r})$ are arbitrary scalar and exchange potentials and $\vec{\tau}$ is the Pauli spin-matrix vector. In the local-spin-density approximation[10] (LSDA) of SDFT,

$\vec{\Delta}(\vec{r}) = \Delta_0(n(\vec{r}), m(\vec{r}))\hat{m}(\vec{r})$ where \hat{m} is a unit vector, $\vec{m} = m\hat{m}$ is the total spin-density at \vec{r} obtained in equilibrium by summing over all occupied orbitals, and the magnitude of the exchange field ($\Delta_0(n, m)$) is the quasiparticle spin-splitting of a polarized uniform electron gas. The spin-density contribution from a single orbital Ψ_α is $\vec{s}_\alpha(\vec{r}) = \Psi_\alpha^\dagger(\vec{r}) \vec{\tau} \Psi_\alpha(\vec{r})/2$. The time-dependent quasiparticle Schroedinger equation therefore implies that

$$\frac{ds_{\alpha,j}(\vec{r})}{dt} = \nabla_i J_{\alpha,j}^i(\vec{r}) + \frac{1}{\hbar} \left[\vec{\Delta} \times \vec{s}_\alpha(\vec{r}) \right]_j \quad (2)$$

where the spin current tensor for orbital α is defined by,

$$J_{\alpha,j}^i(\vec{r}) = \frac{1}{2m} \text{Im} \left(\Psi_\alpha^\dagger(\vec{r}) \tau_j \nabla^i \Psi_\alpha(\vec{r}) \right). \quad (3)$$

This equation exhibits the separate contributions to individual quasiparticle spin dynamics from convective spin flow, the source of the conservative term, and precession around the exchange field $\vec{\Delta}$. Both sides of Eq.[2] vanish when the quasiparticle spinor solves a time-independent Schroedinger equation.

Collective Magnetization Dynamics: The time-dependence of the total magnetization is obtained by summing Eq.[2] over all occupied orbitals.

$$\frac{dm_j(\vec{r})}{dt} = \sum_\alpha \nabla_i J_{\alpha,j}^i(\vec{r}) + \frac{1}{\hbar} \left[\vec{\Delta} \times \vec{m}(\vec{r}) \right]_j \quad (4)$$

where $J_{\alpha,j}^i$ is the contribution to the spin-current from orbital α . The main point we wish to make here is that (in the LSDA) $\vec{\Delta}$ is proportional to \vec{m} at each point in space-time so that the second term on the right vanishes. The collective magnetization dynamics[13] is driven not by the large effective fields seen by the quasiparticles, but by external and demagnetization fields and spin-orbit coupling effects that have been neglected to this point in the discussion, and by the divergence of the collective spin-current[14] in the first term. A complete description of magnetization dynamics would require that the neglected terms be included, and that damping due to magnetophonon and other couplings be recognized. In practice, thin film magnetization dynamics can usually be successfully described using a partially phenomenological *micromagnetic* theory approach[15] in which the long-wavelength limit of the microscopic physics is represented by a small number of material parameters that specify magnetic anisotropy, stiffness, and damping. We adopt that pragmatic approach here, replacing the microscopic Eq.[4] by the phenomenological Landau-Lifshitz equation

$$\frac{\partial \hat{m}}{\partial t} = \hat{m} \times \vec{\Delta}^C + \alpha \hat{m} \times \frac{\partial \hat{m}}{\partial t}, \quad (5)$$

where α is the damping parameter,

$$\vec{\Delta}^C(\vec{r}) \equiv \frac{\delta E_{MM}[\hat{m}]}{\delta \hat{m}(\vec{r})} \quad (6)$$

is the effective field that drives the long-wavelength collective dynamics of an electrically isolated sample, and $E_{MM}[\hat{m}]$ is the micromagnetic energy functional.

Spin-Transfer: When current flows through a ferromagnet, the transport orbitals are few in number and make a negligibly small contribution to the magnitude of the magnetization. In an inhomogeneous magnetic system, however, they can make an important contribution to the exchange field $\vec{\Delta}$ as we now explain. Because $\vec{\Delta}$ is much larger than $\vec{\Delta}^C$, slow collective dynamics can be ignored in the transport theory. Our approach to spin-transfer is based on a scattering theory formulation[11] in which properties of interest can be expressed in terms of scattering solutions of the time-independent Schroedinger equation defined by the instantaneous value of $\vec{\Delta}$. Transport electrons will in general make a contribution to the spin-density that is small but perpendicular to the magnetization[16]. We define this transport contribution to the spin-density as \vec{m}^{tr} . Because it is perpendicular to the magnetization, its contribution to the exchange-field experienced by all quasiparticles

$$\vec{\Delta}^{\text{tr}} = \Delta_0(n, m) \frac{\vec{m}^{\text{tr}}}{m} \quad (7)$$

produces a spin-torque that can be comparable to that produced by $\vec{\Delta}^C$. It follows that the influence of a transport current on magnetization dynamics is captured by replacing $\vec{\Delta}^C$ in Eq.[5] by $\vec{\Delta}^C + \vec{\Delta}^{\text{tr}}$. This proposal is the central idea of our paper.

Our proposal can be related to the present approach in which spin-transfer is computed from spin current fluxes. In the absence of spin-orbit coupling, summing over all transport orbitals and applying Eq.[2] implies a relationship between the transport magnetization and the transport spin currents:

$$\left[\vec{\Delta}(\vec{r}) \times \vec{m}^{\text{tr}}(\vec{r}) \right]_j = -\hbar \nabla_i J_j^{\text{tr},i}(\vec{r}) \quad (8)$$

where $J_j^{\text{tr},i}$ is the spin-current tensor summed over all transport orbitals. Note that the net spin current flux through any small volume is always perpendicular to the magnetization. It follows from Eq.[8] that

$$\vec{\Delta}^{\text{tr}}(\vec{r}) = \frac{\nabla_i \vec{J}^{\text{tr},i}(\vec{r}) \times \hat{m}}{m}. \quad (9)$$

When Eq.[9] is inserted in Eq.[5] it implies a contribution to the local rate of spin-density change in any small volume proportional to the net flux of spin current into that volume; in other words it implies that the *bookkeeping* theory of spin-transfer applies locally, a property that can be traced in this instance to the LSDA of SDFT. This observation helps explain why a simple spin-transfer argument [17] is able to account for the influence of a current on spin-waves in a homogeneous ferromagnet [18]. When spin-orbit interactions are present, Eq.[9] is no longer valid.

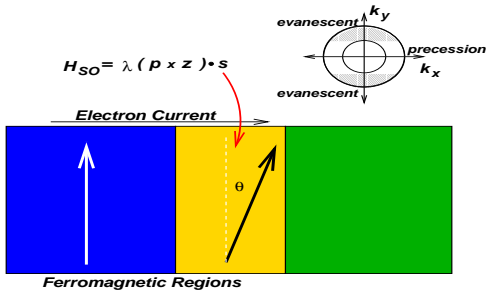


FIG. 2: Toy model described in the text, a 2DEG with ferromagnetic regions. In our calculations we apply periodic boundary conditions in the transverse \hat{y} direction. A spin-transfer torque is present when the two magnetization directions are not aligned. The inset shows the Fermi surfaces of the two ferromagnets in which are identical in the absence of spin-orbit coupling and indicates the transverse channel k_y range over which one of the two Schrodinger equation solutions is an evanescent spinor. The Schrodinger equation solutions for electrons incident from $x \rightarrow -\infty$ can be solved by elementary but tedious calculations in which the spinors and their derivatives are required satisfy appropriate continuity conditions at the interfaces.

Eqs.[5] and [7] provide explicit expressions for the effective magnetic fields that drive magnetization precession at each point in space and time. Using these equations it is possible to explore the consequences of spatial variation in spin-transfer torque magnitude and direction, and of spin-orbit interactions. These have a dominant importance in ferromagnetic semiconductors [19]. *Toy-Model Calculation:* We illustrate our theory by evaluating $\vec{m}^{\text{tr}}(\vec{r})$ for a toy model containing a ferromagnetic two-dimensional electron system with Rashba spin-orbit interactions. The model system, illustrated in Fig.[2], is intended to capture key features of the spin-transfer effect. We take the width of the pinned magnet to infinity, neglect the paramagnetic spacer that is required in practice to eliminate exchange coupling between the two magnets, and assume for simplicity that there is no band offset between the two ferromagnets and that the two exchange fields are equal in magnitude. Current flows from the pinned magnet, through the free magnet, into a paramagnetic metal that functions as a load. The spin-orbit interaction is assumed to be confined to the free magnet region [20] For this model we evaluated $\vec{m}^{\text{tr}}(\vec{r})$ in a current-carrying system using the Landauer-Büttiker approach [11]. In the linear response regime this requires that the Schrodinger equation be solved for electrons incident from the left at the Fermi energy in all transverse channels.

It is helpful at this point to make contact with the usual description of spin-transfer. In its simplest version, spin-transfer theory assumes complete transfer, *i.e.* that the incoming current is spin-aligned in the fixed magnet direction and the outgoing current is spin-aligned in the free magnet direction. To the extent that the complete

transfer assumption is valid, the torque is in the plane defined by the two magnetization orientations, which we refer to as the transfer plane. Microscopically [9] the component of the outgoing current perpendicular to the transfer plane is expected to be very small because of interference between precessing magnetizations in different channels.

It follows from Eq.[7] that the spatially averaged spin orientation of the transport electrons is should be approximately perpendicular to the transfer plane. It can be verified that this is indeed true by directly evaluating $\vec{m}^{\text{tr}}(\vec{r})$. This simple intuitive argument is not exact, however. In particular, the incoming spin current is not necessarily polarized along the pinned magnet magnetization, because of interference between incident and reflected quasiparticle waves that complicate the spin-transfer torques and also because it fails to account for electrons that are described by spinors with evanescent components. (See the inset of Fig.[2]). In any microscopic calculation these effects and others conspire to produce a relatively small component of the torque that is perpendicular to the transfer plane, and correspondingly to a component of $\vec{m}^{\text{tr}}(\vec{r})$ that is in the transfer plane.

In Fig.[3(a)] we plot values of $\vec{m}^{\text{tr}}(\vec{r})$ per unit current averaged over the free magnet space as a function of the angle between the two magnetization orientations, in the case without spin-orbit interaction. We have taken the free magnet orientation to be the \hat{z} direction and the pinned magnet to be in the $\hat{z} - \hat{x}$ plane with polar angle θ .

When spin-orbit interactions are included, the strength of the spin-transfer torque must be evaluated using the transport spin densities. The *bookkeeping* argument, based on total spin conservation, is no longer valid. The quasiparticle spins not only are no longer conserved due to momentum-dependent effective magnetic fields that represent spin-orbit coupling. As we see in Fig.[3], the spin-transfer effect is not only reduced in magnitude but its dependence on θ no longer approximates the simple complete transfer expression. A measure of how the effect is destroyed by the spin-orbit interaction is given by the magnitude of the spin transfer efficiency g_{ST} , defined as the value of the in plane torque per unit current at the optimum geometry, $\theta = \pi/2$. In Fig.[3(c)] we show the efficiency as a function of the spin orbit interaction strength. We see that when the spin-orbit interaction strength is comparable to the exchange spin splitting the effect is strongly reduced except for the case of extremely thin layers. In conclusion we have presented a formalism that allow us to evaluate the interplay between transport currents and magnetization dynamics in very general circumstances. This formalism can address open issues in transport theory including the possible importance of incoherent nanomagnet magnetization dynamics, and the influence of the spin-orbit interactions that are expected to be most important in diluted magnetic semiconductor ferromagnets. This work was supported by the Welch

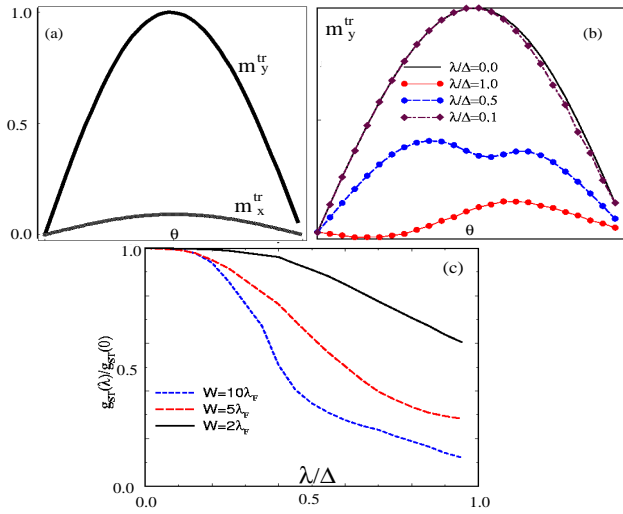


FIG. 3: (a) Transport spin density per unit current in the case without spin-orbit interaction. m_y^{tr} is the component perpendicular to the transfer plane while m_x^{tr} is the smaller component in the transfer plane that is contributed by evanescent spinors. Both components are normalized to the maximum m_y^{tr} which occurs for $\theta = \pi/2$. (b) Non-equilibrium spin density per unit current perpendicular to the transfer plane for different spin-orbit interaction strengths. It follows that from these results that the spin-transfer torque is reduced in efficiently and altered in angle dependence by spin-orbit interactions. (c) Spin transfer efficiency, g_{ST} , normalized by the ST efficiency in the absence of spin-orbit coupling, as a function of the spin-orbit strength, for several widths of the free magnet. The spin-transfer effect becomes weak when the spin-orbit splitting is comparable with the exchange splitting.

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